Final Exam , MTH 221 , Fall 2010

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(i) (10 points)

a. Let A and B be two 4×4 matrices with $det(A) = -8$ and $det(B) = 1/2$. Find $det(2A^{-2}B^4)$.

b. Evaluate the determinant of $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ -1 & 1 & 1 & -1 & 1 \end{bmatrix}$.

(ii) (8 points) Determine the value(s) of a such that the points $(1, 0, 5), (1, 2, 4), (1, 4, a)$ are dependent.

(iii) (10 points) Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$. Find a diagonal matrix D and an invertible matrix Q such that $A = QDQ^{-1}$. Find A^{24}.

(iv) (7 points) Suppose A is a 3×3 matrix and $A - \alpha I_3 = (1-\alpha)(2-\alpha)^2$. Given $N(A-I) = span\{(1,2,0)\}$ and $N(A-2I) = span\{(2,0,3)\}$. Is A diagnolizable? Explain.

(v) (10 points) Solve the following system of equations

$$\begin{bmatrix} 1 & 0 & -1 \\ -3 & -1 & 0 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}.$$

(vi) (10 points) The augmented matrix of a linear system has the form $\left[\begin{array}{ccc|c} 1 & 2 & 1 & a \\ 2 & 3 & -2 & b \\ -1 & -1 & 3 & c \end{array}\right]$

a. For what values of a, b, and c will the system be consistent? (you may write your answer as one equation in terms of $a.b$ and c).

b. When the above system is consistent, does the system have a unique solution or infinitely many solutions? explain.

(vii) (15 points)

a. In each question below a vector space V is given, together with a subset $W \subseteq V$. In each case state (with justification) whether or not W is a sub*space* of V.

i. $V = R^2,\ W = \left\{(x,y) \in R^2 |\ x \geq 0 \text{ and } y \geq 0\right\}.$

ii. $V = R_{2\times 2},\ W = \{A \in R_{2\times 2} | Rank(A) \leq 1\}$

b. Find a basis for the subspace W where

$$W = \left\{ \begin{bmatrix} a - b + 3c & 4a + 3b - 9c & 2a \\ 8a + 2b - 6c & 5a & 0 \end{bmatrix} \mid a, b, c \in R \right\}$$

(viii) (10 points) Let $A = \begin{bmatrix} 2 & -4 & 0 & -6 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ -1 & 2 & 0 & 3 & 6 \end{bmatrix}$. If $T : R^5 \rightarrow R^3$ is defined by $T((a_1, a_2, a_3, a_4, a_5)) = A \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}$

a. Find a basis for the range of T

b. Find $Ker(T)$ and Write $Ker(T)$ as a span

(ix) (10 points) The linear transformation $T : P_3 \to R$ is given by

$$T(p(x)) = \int_{-1}^{1} p(x)dx$$

a. Find a $p(x)$ in P_3 so that $T(p(x)) = 2011$.

b. Find $Ker(T)$ and write Ker(T) as a span.

c. Is T one to one?explain

(x) (10 points) Let $M = span\{(0,0,1,1),(1,0,1,1),(1,-1,1,0)\}$. Find an orthogonal basis for M.

(**Bonus problem, 5 points**)(You may answer it on the back) Let A_n be the 2×2 matrix defined by

$$A_n = \begin{bmatrix} 1-n & -n \\ n & 1+n \end{bmatrix}.$$

a. Prove that $A_n A_m = A_{n+m}$.

b. If $B = \begin{bmatrix} -1 & -2 \\ 2 & 3 \end{bmatrix}$ compute B^{2011}.

Faculty information

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